TAX PYRAMIDING STUDY METHODOLOGY AND DATA SOURCES

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The tax pyramiding estimates in this study follow the methodology outlined in the 2002 Washington State Tax Structure Study Committee's final report.[1] In this section we outline the theory of measuring tax pyramiding and the data sources used.

I. Theory of Input-Output

We begin with an input-output view of the Washington State economy. The economy is divided into *n* industries, each one buying inputs, adding value to them, and selling the resulting output to consumers, governments and the rest of the world.

Two relationships hold in this framework. First, the total value of each industry's output equals the amount spent on inputs plus any value added in the production process. Second, each industry's total output must equal the amount other industries buy from them as inputs plus the amount they sell as final output to consumers, governments and the rest of the world.

For industry one, we can write these relationships mathematically:

(1) $$Y_1 = x_{11} + x_{12} + ... + x_{1n} + V_1$$

Where:

Y_1 = Total output from industry one;

$x_{11}, x_{12}, ..., x_{1n}$ = Intermediate inputs purchased by industry one from industries one through *n*; and

V_1 = The value added by industry one.

(2) $$Y_1 = x_{11} + x_{21} + ... + x_{n1} + D_1$$

Where:

Y_1 = Total output from industry one;

$x_{11}, x_{21}, ..., x_{n1}$ = The amount of output from industry one purchased as intermediate inputs by industries one through *n*; and

D_1 = The amount of industry one's output sold as final demand.

[1] *Tax Alternatives for Washington State: A Report to the Legislature*, Appendix C-12, “"Effective Tax Rates on Value Added and the Degree of Pyramiding in the Gross Receipts Tax” (November 2002). Available online at http://dor.wa.gov/content/aboutus/statisticsandreports/wataxstudy/Final_Report.htm.

These relationships can also be summarized as a standard input-output table. In table (3) below, summing down the columns corresponds to equation (1), while summing across the rows corresponds to equation (2):

$$(3)\quad \begin{matrix} x_{11} & x_{21} & \cdots & x_{n1} & D_1 \\ x_{12} & x_{22} & \cdots & x_{n2} & D_2 \\ \vdots & & \ddots & \vdots & \vdots \\ x_{1n} & x_{2n} & \cdots & x_{nn} & D_n \\ V_1 & V_2 & \cdots & V_n & \end{matrix}$$

II. The Leontief Model

To measure tax pyramiding, table (3) can be manipulated into a Leontief input-output model as follows. Reading down the columns of the table, the sum of inputs purchased by each column industry plus their value added is equal to total output for that industry. Put differently,

$$(4)\qquad Y_1 = \sum_{j=1}^{n} x_{1j} + V_1$$

To simplify the algebra, we define a new number a_{ij}. Let a_{ij} equal the share of each row industry *j*'s total output purchased as an intermediate input by each column industry *i*. That is, let:

$$(5)\qquad a_{ij} = \frac{x_{ij}}{Y_j}$$

This is known as the "input coefficient" between industries *i* and *j*, and will generally be a fraction between zero and one. If industry *j*'s total output represents a pie, a_{ij} represents the slice purchased by industry *i* as inputs. For example, if $a_{12} = 0.2$ column industry one buys 20 percent of row industry two's output.

Define a new matrix *A*, which simply collects the a_{ij} coefficients for all industries in the economy into an *n x n* matrix. To calculate *A*, we divide each *x* entry in the input-output table (3) by its row sum. The result is the following:

$$(6)\qquad A = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \vdots & \ddots & & \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{bmatrix}$$

We can now substitute a_{ij} into equation (4) and rewrite it as follows:

(7) $$Y_1 = \sum_{j=1}^{n} a_{1j} \cdot Y_j + V_1$$

This equation (7) only shows the relationship between inputs and outputs for industry one. We can generalize it for all industries by expressing equation (7) in matrix notation as follows:

(8) $$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \ddots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} + \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

In equation (8), the *n* x *n* matrix of a_{ij}'s on the right-hand side is simply the transpose of the *A* matrix defined above. Label this *A'*. Also, by labeling the *n x 1* vector of *Y*'s as *Y*, and the *n x 1* vector of *V*'s as *V*, we can rewrite equation (8) more simply as:

(9) $$Y = A' \cdot Y + V$$

Solving this for *Y*, we get:[2]

$$Y - A' \cdot Y = V$$

$$(I - A') \cdot Y = V$$

(10) $$Y = (I - A')^{-1} \cdot V$$

Equation (10) is the basic input-output model of the economy relating total economic output to the input coefficients between firms and business' value added. This is the standard "Leontief model" of the economy, named after economist Wassily Leontief.

The economy described in (10) ignores the effect of taxes. The next step is to create an alternative model that includes a gross-receipts-style B&O tax. This will allow us to compare the two models and isolate the impact of the tax.

To add B&O taxes to the model, recall equation (6) which says the output of industry one is equal to its input purchases plus value added:

(6) $$Y_1 = \sum_{j=1}^{n} a_{1j} \cdot Y_j + V_1$$

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[2] In equation (10), *I* represents an *n* x *n* identity matrix with 1's along the diagonal and zeros elsewhere.

Gross receipts taxes apply to sales of intermediate inputs as well as final outputs. Assuming the tax is fully passed forward to final consumers, a B&O tax is equivalent to multiplying the right-hand side of equation (6) by $(1+t_j)$, where t_j is the effective B&O tax rate on industry *j*. Equation (6) then becomes:

$$(11) \qquad \hat{Y}_1 = \left(\sum_{j=1}^{n} a_{1j} \cdot Y_j + V_1 \right) \cdot (1+t_j)$$

In equation (11), industry output *Y* has been relabeled $\hat{Y}$ to denote that it includes the built-in gross receipts tax. To finish the model, we define a matrix *T*, which is an *n* x *n* matrix with (1 + industry effective B&O tax rate) along the diagonal and zeros elsewhere:

$$(12) \qquad T = \begin{bmatrix} (1+t_1) & 0 & \cdots & 0 \\ 0 & (1+t_2) & & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & (1+t_n) \end{bmatrix}$$

Using the simplified notation from equation (9) above, we rewrite equation (11) as the following:

$$(13) \qquad \hat{Y} = T \cdot A' \cdot Y + T \cdot V$$

Solving for $\hat{Y}$, we get:

$$\hat{Y} - T \cdot A' \cdot Y = T \cdot V$$

$$(I - T \cdot A') \cdot \hat{Y} = T \cdot V$$

$$(14) \qquad \hat{Y} = (I - T \cdot A')^{-1} \cdot T \cdot V$$

Equation (15) provides an input-output model that includes a gross receipts tax. These two models from equations (10) and (14) can then be compared to estimate gross receipts tax pyramiding:

(10) No Tax: $Y = (I - A')^{-1} \cdot V$

(14) With Tax: $\hat{Y} = (I - T \cdot A')^{-1} \cdot T \cdot V$

The difference between each industry's total output in the two models represents the embedded forward-passed tax burden from the B&O tax. To measure pyramiding, we first express the tax burden for each industry as a percentage of their value added. This yields each industry's effective tax rate on value added. We then divide this figure by the industry's effective gross receipts tax rate using actual tax collections and bases.

If the result is one, there is no tax pyramiding. In that case, the industry's B&O tax liability would equal their effective B&O tax rate implying the tax was paid only once. If the result is greater than one the B&O tax causes pyramiding. Following the 2002 Washington State Tax Structure Committee study we label this the "degree of pyramiding" of the tax.

III. Data Sources

The basis for the input-output model in this study is the 2002 Washington State Input-Output table released in May 2008.[3] The table is a "square" input-output table that divides the state's economy into 51 sectors—50 domestic industries and one industry for imports from the rest of the world.

To generate the *T* matrix of industry effective B&O tax rates, we use actual tax collections and tax base data for 2007 from the Washington State Department of Revenue[4]. Effective tax rates for the NAICS industry groupings reported by the Department of Revenue were then mapped to the 51 industries in the input-output table. A complete crosswalk of NAICS categories to the 51 input-output industries is available from the authors upon request.

[3] Washington State Office of Financial Management (OFM), "The 2002 Washington Input-Output Model" (May 2008). Available online at http://www.ofm.wa.gov/economy/io/2002/default.asp.

[4] Washington State Department of Revenue, *Quarterly Business Review*, "Table 5: Business and Occupation Tax: Gross Income, Taxable Income and Tax Due Statewide Amounts By Industry (NAICS) Calendar 2007". Available online at http://dor.wa.gov/content/aboutus/statisticsandreports/TID/ResultsTable5.aspx?Period=2007AN&Type=naics&Format=HTML.